

Probing freeze-out conditions in heavy ion collisions with moments of charge fluctuations

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July 15, 2010

Abstract

We calculate the first four moments of baryon number, electric charge and strangeness fluctuations within the hadron resonance gas model. Different moments and their ratios as well as skewness and kurtosis are evaluated on the phenomenologically determined freeze-out curve in the temperature, baryon chemical potential plane. The model results and its predictions as well as relations between different moments are compared with the first data on net proton fluctuations in Au-Au collisions obtained at RHIC by the STAR Collaboration. We find good agreement between the model calculations and experimental results. We also point out that higher order moments should be more sensitive to critical behavior and will also distinguish hadron resonance gas model calculations from results obtained from lattice QCD.

CERN-PH-TH-2010-161

1 Introduction

One of the major goals in theoretical and experimental studies of the thermodynamics of strongly interacting matter is to explore the QCD phase diagram at non-zero temperature (T) and non-zero baryon chemical potential (μ_B) [1]. A central target of interest is the 'critical point' - a second order phase transition point, that has been postulated to exist in the T - μ_B phase diagram [2, 3]. Although its existence is not yet firmly established, its presence would result in large correlation lengths, *i.e.* large fluctuations in various thermodynamic quantities. Remnants of these large fluctuations may become accessible in heavy ion collisions through an event-by-event analysis of fluctuations [4] in various channels of hadron quantum numbers (charges), eg. baryon number (B), electric charge (Q) and strangeness (S). In fact, at vanishing baryon chemical potential it has been shown that moments of charge fluctuations are sensitive indicators for the occurrence of a transition from hadronic to quark-gluon matter [5]. They drastically change during the transition.

When comparing theoretical equilibrium calculations of charge fluctuations with experimental results from heavy ion collisions, the crucial question is, whether at the time of hadronization the thermal system generated in these collisions has kept memory of the plasma phase or the expansion period during which it may have passed by a critical point [6]. If so, this may lead to an enhancement of fluctuations over ordinary thermal effects at freeze-out. On the other hand, it is well known that basic features of hadronization in heavy ion collisions are well described in terms of the hadron resonance gas model (HRG) [7, 8]. The analysis of experimental data on the production cross sections of various hadrons in heavy ion collisions shows astonishingly good agreement with corresponding thermal abundances calculated in a HRG model at appropriately chosen temperature and chemical potential [9, 10]. Abundances of strange and non-strange mesons and baryons are consistently described by freeze-out temperatures and baryon chemical potentials that are a function of collision energy only.

If some memory of large correlation lengths persists in the thermal medium at time of freeze-out this must be reflected in higher moments of charge distributions. These moments are more sensitive to large correlation lengths and thus relax more slowly to their true equilibrium values at the time of freeze-out [11]. It thus is an interesting question to test whether also more detailed information on the thermal distribution of hadron species, predicted by the HRG model, can be confirmed experimentally. In fact, one may consider HRG model results on moments of charge fluctuations as a theoretical baseline prediction for the currently ongoing low energy heavy ion runs at RHIC [12] and future studies of charge fluctuations at the LHC. Any deviations from HRG model predictions would constitute evi-

dence for new phenomena at the time of hadronization that have not been seen in experiments so far.

Of course, eventually thermodynamics at freeze-out should be described by thermal QCD, eg. through lattice calculations. A direct comparison of experimental and HRG model results on higher moments of charge fluctuations with lattice calculations in the hadronic phase is possible [13], but is still difficult as so far most lattice calculations are performed with staggered fermions on rather coarse lattices. They need to be performed closer to the continuum limit to reproduce the correct hadron spectrum [14, 15]. Fortunately the calculation of ratios of moments is less sensitive to such cut-off effects [5, 13, 16]. At present, it seems that lattice QCD calculations of ratios of moments of charge fluctuations, performed at non-vanishing chemical potential by using a Taylor expansion of thermodynamic observables [17, 18], are in good agreement with HRG model calculations for temperatures below the transition temperature. We will consider this issue in more detail in Section 5 and indicate when such agreement may break down.

In the following we will discuss the dependence of higher moments of fluctuations of baryon number, electric charge and strangeness on the collision energy. In particular, we will calculate ratios of quartic and quadratic (kurtosis), cubic and quadratic (skewness) as well as quadratic charge fluctuations normalized to their mean value along a phenomenologically determined freeze-out curve in heavy ion collisions. We also discuss correlations of different charges, eg. the correlation of baryon number and electric charge normalized to the squared baryon number fluctuations.

In the next section we will summarize basic results on the parametrization of the freeze-out curve in heavy ion collisions. In section 3 we introduce moments of charge fluctuations and discuss their calculation in the HRG model. In section 4 we compare the HRG model results with recent data on net proton number fluctuations in heavy ion collisions at RHIC energies obtained by the STAR Collaboration [19] and discuss additional fluctuation observables that may be analyzed to further characterize thermal conditions at freeze-out. Section 5 is devoted to a comparison of the HRG model and lattice QCD calculations. We give our conclusions in section 6.

2 Freeze-out conditions in heavy ion collisions

Abundances of strange and non-strange mesons and baryons produced in heavy ion collisions in a wide range of collision energies are consistently described by freeze-out temperatures and baryon chemical potentials that are a function of collision energy only. In fact, the freeze-out curve $T(\mu_B)$ in the T - μ_B plane and

X	d [GeV]	e [GeV ⁻¹]
B	1.308(28)	0.273 (8)
S	0.214	0.161
Q	0.0211	0.106

Table 1: Parametrization of chemical potentials μ_X along the freeze-out curve using the ansatz given in Eq. 2.

the dependence of the baryon chemical potential on the center of mass energy in nucleus-nucleus collisions can be parametrized by simple functions [20]

$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4, \quad (1)$$

where $a = (0.166 \pm 0.002)$ GeV, $b = (0.139 \pm 0.016)$ GeV⁻¹, $c = (0.053 \pm 0.021)$ GeV⁻³ and

$$\mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}}, \quad (2)$$

with $d = (1.308 \pm 0.028)$ GeV and $e = (0.273 \pm 0.008)$ GeV⁻¹. This parametrization agrees with the phenomenological freeze out condition of fixed energy per particle of about 1 GeV [21].

The energy dependence of strange and electric charge chemical potentials are obtained from the HRG model by demanding strangeness neutrality and isospin asymmetry in the initial state of Au-Au collisions. They can be parameterized in the same way as it has been done for the baryon chemical potential in Eq. 2. The corresponding fit parameters are given in Table 1 and are shown in the left hand part of Fig. 1. The right hand part of this figure shows the variation of baryon number density, $n_B = \langle N_B \rangle / V$, and electric charge density, $n_Q = \langle N_Q \rangle / V$, on the freeze-out curve. The strangeness density, $n_S = \langle N_S \rangle / V$, vanishes due to the imposed strangeness neutrality condition. We note that the ratio of baryon to strangeness chemical potential on the freeze-out curve shows only a weak dependence on the collision energy,

$$\frac{\mu_S}{\mu_B} \simeq 0.164 + 0.018\sqrt{s_{NN}}, \quad (3)$$

which is in agreement with findings on freeze-out conditions at RHIC [22].

Current results from lattice calculations [14, 15, 23] indicate that at $\mu_B \simeq 0$ the transition temperature, within errors, coincides with the freeze-out temperature extracted at the highest RHIC energy. However, although firm statements from lattice QCD need to wait for calculations closer to the continuum limit, it seems

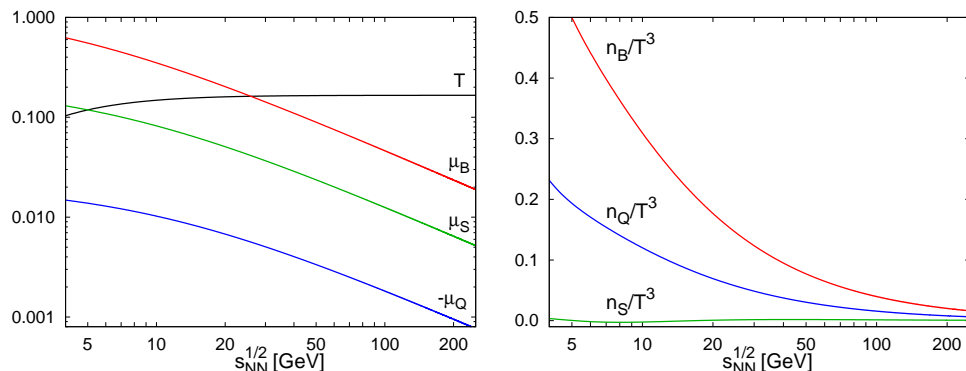


Figure 1: The energy dependence of temperature T and chemical potentials for baryon number μ_B , electric charge μ_Q and strangeness μ_S along the chemical freeze-out curve in units of GeV (left). The right hand figure shows the variation of baryon number density n_B , electric charge density n_Q and strangeness density n_S along the freeze-out curve.

that for $\mu_B > 0$ the curvature of the transition line $T(\mu_B)$ for $\mu_B > 0$ is distinct from the freeze-out curve [24]. This may indicate that at larger μ_B the QCD transition and the hadronic freeze-out in heavy ion collisions are separated. The latter seems to occur at lower temperatures.

If this separation of transition and freeze-out curves is indeed confirmed and/or hadronic fluctuations at freeze-out lose all memory of the passage through a possible transition region, then not only particle yields but also charge fluctuations should be characterized by thermal freeze-out conditions in the hadronic phase and may be well described by the HRG model.

In the next section we will work out ratios of moments of various conserved charges on the freeze-out curve using the HRG model. In particular, we will calculate ratios of quadratic, cubic and quartic fluctuations of baryon number.

3 Fluctuations in the HRG model

The basic quantity that describes thermodynamics at non-vanishing chemical potential is the pressure. In the grand-canonical ensemble with baryon number, strangeness and electric charge conservation it is obtained from the logarithm of the partition function as

$$p(T, \mu_B, \mu_Q, \mu_S) = \lim_{V \rightarrow \infty} \frac{T}{V} \ln Z(T, \mu_B, \mu_Q, \mu_S, V) \quad . \quad (4)$$

In the HRG model the partition function contains all relevant degrees of freedom of the confined, strongly interacting matter and implicitly includes interactions that result in resonance formation. The logarithm of the partition function in the HRG is obtained as a sum over all stable hadrons and resonances and their anti-particles, which can be separated into contributions from baryons and mesons,

$$\ln Z(T, \mu_B, \mu_Q, \mu_S) = \sum_{i \in \text{mesons}} \ln Z_i^+(T, \mu_Q, \mu_S) + \sum_{i \in \text{baryons}} \ln Z_i^-(T, \mu_B, \mu_Q, \mu_S) \quad (5)$$

The contribution of each particle species of mass m_i , spin degeneracy factor g_i , carrying baryon number B_i , electric charge Q_i and strangeness S_i is expressed as

$$\ln Z_i^\pm(T, V, \vec{\mu}) = \frac{VT}{2\pi^2} g_i m_i^2 \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k^2} K_2(km_i/T) \exp(k\vec{c}_i \vec{\mu}/T) , \quad (6)$$

where $\vec{c}_i = (B_i, Q_i, S_i)$, $\vec{\mu} = (\mu_B, \mu_Q, \mu_S)$ and $K_2(x)$ is the modified Bessel function. The upper and lower signs are for bosons and fermions, respectively. The first term in Eq. 6 corresponds to the Boltzmann approximation.

The partition function (Eq. 5) together with Eq. 6 provides the basis for the description of the thermodynamics of a system composed of hadrons and resonances being in thermal and chemical equilibrium. In the following we will focus on HRG model predictions on fluctuations of conserved charges and their higher moments.

Fluctuations of the net charge N_q and its higher moments are obtained from derivatives of the logarithm of the partition function with respect to the corresponding chemical potential. These derivatives define generalized susceptibilities¹

$$\chi_q^{(n)} = \frac{\partial^n [p(T, \vec{\mu})/T^4]}{\partial (\mu_q/T)^n} . \quad (7)$$

The first derivative, $\chi_q^{(1)}$, is related to the mean value M_q of the net charge N_q ,

$$M_q \equiv \langle N_q \rangle = VT^3 \chi_q^{(1)} , \quad (8)$$

i.e. it is the charge density in units of T^3 , $\chi_q^{(1)} \equiv n_q/T^3$. The second derivative with respect to μ_q/T gives the variance $\sigma_q^2 = \langle (\delta N_q)^2 \rangle$ as

$$\sigma_q^2 = VT^3 \chi_q^{(2)} , \quad (9)$$

¹Note that we introduce here dimensionless susceptibilities. This notation differs from that used in Ref. [19] by a factor T^{n-4} .

with $\delta N_q = N_q - \langle N_q \rangle$. The third $\chi_q^{(3)}$ and the fourth $\chi_q^{(4)}$ order moments can be expressed in terms of δN_q as

$$\langle (\delta N_q)^3 \rangle = VT^3 \chi_q^{(3)}, \quad (10)$$

$$\langle (\delta N_q)^4 \rangle - 3\langle (\delta N_q)^2 \rangle^2 = VT^3 \chi_q^{(4)}. \quad (11)$$

We also introduce skewness (S_q) and kurtosis (κ_q), which are generally used to characterize the shape of statistical distributions,

$$S_q \equiv \frac{\langle (\delta N_q)^3 \rangle}{\sigma_q^3}, \quad \kappa_q \equiv \frac{\langle (\delta N_q)^4 \rangle}{\sigma_q^4} - 3. \quad (12)$$

Using Eqs. 8 to 12 we may relate mean values, fluctuations, skewness and kurtosis to the generalized susceptibilities introduced in Eq. 7,

$$\frac{\sigma_q^2}{M_q} = \frac{\chi_q^{(2)}}{\chi_q^{(1)}}, \quad S_q \sigma_q = \frac{\chi_q^{(3)}}{\chi_q^{(2)}}, \quad \kappa_q \sigma_q^2 = \frac{\chi_q^{(4)}}{\chi_q^{(2)}}. \quad (13)$$

Finally we also consider correlations of charges, which can be obtained as mixed derivatives of the pressure with respect to chemical potentials for charge X and Y , respectively,

$$\chi_{XY}^{(nm)} = \frac{\partial^{n+m} p(T, \vec{\mu}) / T^4}{\partial (\mu_X / T)^n \partial (\mu_Y / T)^m}. \quad (14)$$

We will discuss the behavior of the ratios $\chi_{BS}^{(1m)} / \chi_B^{(2)}$ and $\chi_{BQ}^{(1m)} / \chi_B^{(2)}$ for $m = 1, 3$ on the freeze-out curve.

The above relations can be used to analyze properties of a thermodynamic system in equilibrium. If the evolution of a heavy ion collision results in an equilibrated state that lost all its memory of the previous evolution and hadronizes according to a hadronic statistical model, e.g. the HRG model, this will be reflected not only in particle yields but also in various moments of charge fluctuations. As higher moments become increasingly sensitive to large correlation lengths, any deviations from HRG model predictions at freeze-out could indicate that the system memorizes that it went through a region with large correlation lengths.

In the following we compare HRG model predictions with first measurements of the kurtosis, skewness and variance of net proton multiplicity distributions at mid-rapidity for Au+Au collisions at $\sqrt{s_{NN}} = 19.6, 62.4$ and 200 GeV performed by the STAR collaboration [19].

4 Charge fluctuations on the hadronic freeze-out curve

In the Boltzmann approximation, which is a suitable approximation in the parameter range considered, the HRG model provides a simple result for the thermodynamic pressure,

$$\frac{P}{T^4} = \frac{1}{\pi^2} \sum_i d_i (m_i/T)^2 K_2(m_i/T) \cosh[(B_i \mu_B + S_i \mu_S + Q_i \mu_Q)/T] . \quad (15)$$

The contribution of anti-particles is explicitly included in Eq. 15 through the $\cosh[x]$ -term. Thus, the summation is to be taken only over stable hadrons and resonances.

A specific dependence of the pressure given in Eq. 15 on chemical potentials implies definite predictions for ratios of different moments of charge fluctuations. In particular, from Eqs. 7 and 15, it is immediately clear, that ratios of fourth ($\chi_B^{(4)}$) and second ($\chi_B^{(2)}$) order moments of the baryon number fluctuation as well as the ratio of third ($\chi_B^{(3)}$) and first order moments are unity,

$$\frac{\chi_B^{(4)}}{\chi_B^{(2)}} = 1, \quad \frac{\chi_B^{(3)}}{\chi_B^{(1)}} = 1 , \quad (16)$$

irrespective of the values of chemical potentials and temperature. A direct consequence of the above HRG model results is that simple relations hold for the mean net baryon number, kurtosis and skewness,

$$\kappa_B \sigma_B^2 = 1 \quad , \quad \kappa_B M_B = S_B \sigma_B . \quad (17)$$

The first relation is straightforward from Eq. 13, the second is valid since

$$\kappa_B M_B = \frac{\chi_B^{(4)}}{\chi_B^{(2)}} \frac{M_B}{\sigma^2} = \frac{\chi_B^{(4)}}{\chi_B^{(2)}} \frac{\chi_B^{(1)}}{\chi_B^{(2)}} = \frac{\chi_B^{(4)}}{\chi_B^{(2)}} \frac{\chi_B^{(1)}}{\chi_B^{(3)}} \frac{\chi_B^{(3)}}{\chi_B^{(2)}} = S_B \sigma_B . \quad (18)$$

In the HRG model the skewness $S_B \sigma_B$ can be explicitly obtained from

$$S_B \sigma_B = \frac{\sum_{i \in \text{baryons}} d_i (m_i/T)^2 K_2(m_i/T) \sinh[(\mu_B + S_i \mu_S + Q_i \mu_Q)/T]}{\sum_{i \in \text{baryons}} d_i (m_i/T)^2 K_2(m_i/T) \cosh[(\mu_B + S_i \mu_S + Q_i \mu_Q)/T]} . \quad (19)$$

Consequently, for $\mu_S = \mu_Q = 0$ one gets

$$S_B \sigma_B = \tanh(\mu_B/T) . \quad (20)$$

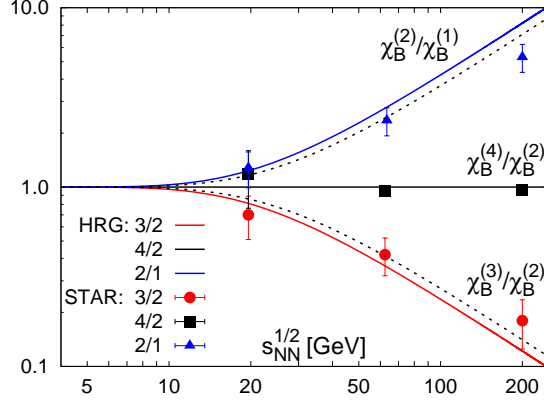


Figure 2: The ratio of quadratic fluctuations and mean net baryon number (σ_B^2/M_B), cubic to quadratic ($S_B\sigma_B$) and quartic to quadratic ($\kappa_B\sigma_B^2$) baryon number fluctuations calculated in the HRG model on the freeze-out curve and compared to results obtained by the STAR collaboration [19]. The dashed curves show the approximate $\tanh(\mu_B/T)$ result for $\kappa_B\sigma_B^2$ and $S_B\sigma$, respectively.

This simple result arises from the fact that in the HRG model only baryons with baryon number $B = 1$ contribute to the various moments.

In heavy ion collisions the strangeness and electric charge chemical potentials are much smaller than μ_B (see Fig. 1). The above relation thus can be considered to be a good estimate of skewness at chemical decoupling. We will show in the following that corrections due to non-vanishing electric charge and strangeness chemical potentials are indeed small for baryon number fluctuations.

4.1 Comparison of the HRG model results on baryon number fluctuations with RHIC data

The relations for skewness and kurtosis summarized in Eqs. (17), (19) and (20) are generic results, expected to hold if thermodynamics is governed by the HRG model. Knowing the energy dependence of thermal parameters along the freeze-out curve (Eqs. 1 and 2) we can directly verify if these particular relations, deduced within the HRG model, are consistent with recent findings of the STAR collaboration, which measured moments of baryon number fluctuations through net-proton number fluctuations [19].

Fig. 2 shows a comparison of the energy dependence of quadratic fluctuations (σ_B^2) normalized to the net baryon number (M_B), skewness $S_B\sigma_B$ and kurtosis $\kappa_B\sigma_B^2$ obtained in the HRG model at chemical freeze-out with the STAR data.

Obviously, the HRG model provides a good description of properties of different moments of net proton number fluctuations measured at RHIC energies. The reason for considering ratios of charge fluctuations rather than absolute values for different moments was, of course, that one is independent of definitions of the interaction volume and also is less sensitive to experimental cuts and systematic errors. Moreover, some of these ratios have an interesting interpretation, like e.g. the ratio $\chi_B^{(4)}/\chi_B^{(2)}$ which directly characterizes the dominant degrees of freedom carrying baryon number [5].

In addition it is, of course, also of interest to understand whether the HRG model can quantitatively describe the energy dependence of the STAR data [19] on the first four moments, i.e. the mean, variance, skewness and kurtosis².

In order to compare the HRG model calculations with the experimental results presented in [19] we note that this analysis only explored fluctuations in a limited phase space. In fact, the data on mean particle yields differ from previous results obtained by the STAR Collaboration [22]. From Ref. [19] one gets: $M_{p-\bar{p}} \simeq 1.75 \pm 0.25$ and $M_{p-\bar{p}} \simeq 3.5 \pm 0.4$ in the central (0-5%) bin of Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV and 62.4 GeV, respectively. These values should be compared with $M_{p-\bar{p}} \simeq 8 \pm 1.8$ and $M_{p-\bar{p}} \simeq 15.4 \pm 2.1$ obtained at mid-rapidity at corresponding energies in [22]. These data differ by a common factor $K \simeq 0.22$. Part of the difference may be attributed to the fact that net proton fluctuation data in Ref. [19] were taken in the restricted transverse momentum range $0.4 < p_T < 0.8$ GeV.

In the HRG model used by us the thermal phase-space of all particles is not restricted. Consequently, in order to compare predictions for different moments of net proton fluctuations with experimental data one needs to rescale its thermal phase space by the above mentioned factor $K \simeq 0.22$. Effectively, this corresponds to rescaling the volume parameter VT^3 appearing for instance in Eq. 8, by the K -factor, although its origin is not necessarily related with a smaller volume of the system at chemical freeze out.

The change of volume with energy on the freeze out line is calculated by comparing data on dN_{π^-}/dy at mid rapidity for different \sqrt{s} with HRG

²Of course, as we have already verified consistency of three ratios with the HRG model calculations, only the energy dependence of one of these observable provides additional information.

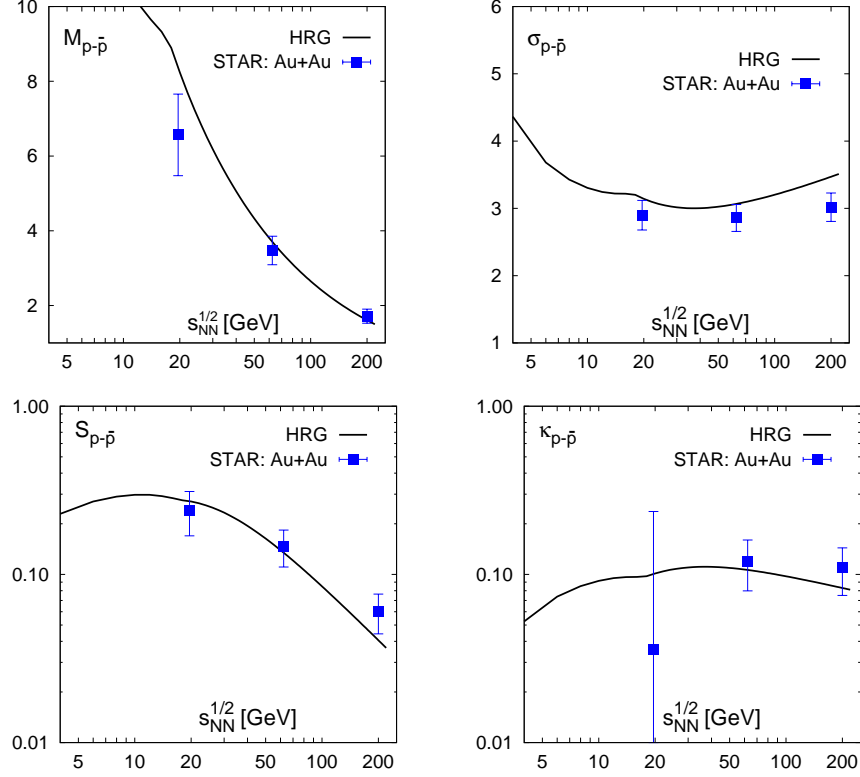


Figure 3: The energy dependence of mean ($M_{p-\bar{p}}$) (top, left), square root of the variance ($\sigma_{p-\bar{p}}$) (top, right), skewness ($S_{p-\bar{p}}$) (bottom, left) and kurtosis ($\kappa_{p-\bar{p}}$) (bottom, right) of net proton number fluctuations. The points are the RHIC results obtained by the STAR collaboration [19]. The lines are calculated in the rescaled (see text) HRG model on the freeze out curve.

model results.³ We then obtain, $V = [dN_{\pi^-}/dy]^{data}/n_{\pi^-}^{HRG}[T, \vec{\mu}]$, where in the HRG model the negatively charged pion density $n_{\pi^-}^{HRG}$ is calculated using the relation between \sqrt{s} and the thermal parameters given in Eqs. 1 and 2. Our results on $V(\sqrt{s})$ extracted in this way are consistent with those obtained recently in Ref. [10].

Fig. 3 (top left) shows the energy dependence of the first moment ($M_{p-\bar{p}}$) of net proton number in the HRG model with a volume parameter, VT^3 , rescaled by the factor $K \simeq 0.22$. One can see in this figure that the HRG model results are consistent with the data.

Taking into account the results for various ratios of moments shown in Fig. 2 it immediately follows that the HRG model will also describe the energy dependence of other moments, *i.e.* variance, skewness and kurtosis. These are also shown in Fig. 3.

The good agreement of HRG model calculations with RHIC data allows us to make predictions for different moments of charge fluctuations covering the energy range of the RHIC low energy scan and the lowest energy for heavy ion collision at the LHC. We summarize the HRG model results at different energies in Table 2.

4.2 Electric charge and strangeness fluctuations

More subtle dependencies on temperature and baryon number arise in the case of electric charge and strangeness fluctuations where multiple charged hadrons get larger weight in higher moments and where meson as well as baryon sectors contribute. This results in characteristic deviations of the kurtosis, more precisely $\kappa_Q \sigma_Q^2 = \chi_Q^{(4)}/\chi_Q^{(2)}$, from unity and also the skewness no longer is simply related to $\tanh(\mu_B/T)$. In the case of electric charge fluctuations we may separate contributions of different charge sectors to the partition function. For instance, for n even, we may then obtain for moments of electric charge fluctuations,

$$\chi_Q^{(n)} = \frac{1}{VT^3} \left(\ln Z_{|Q|=1}(T, \mu_B, \mu_Q, \mu_S) + 2^n \ln Z_{|Q|=2}(T, \mu_B, \mu_Q, \mu_S) \right), \quad (21)$$

where the logarithms of partition functions, $\ln Z_{|Q|}$, are obtained from Eq. 5 by restricting the sums over mesons and baryons to the relevant charge sec-

³For a compilation of heavy ion data on charged pion yields at mid-rapidity at different energies see e.g. Ref. [8].

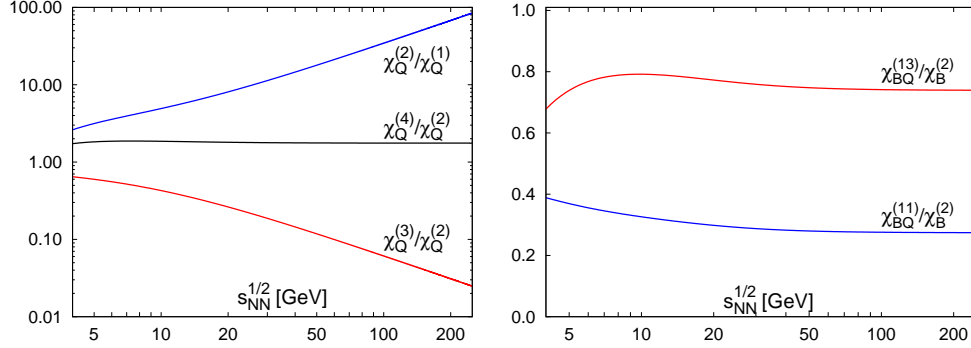


Figure 4: The ratio of moments of electric charge fluctuations on the freeze-out curve (left) and correlations between baryon number and electric charge fluctuations (right).

tors. From this it is obvious that deviations of $\kappa_Q \sigma_Q^2 = \chi_Q^{(4)} / \chi_Q^{(2)}$ from unity only arises from contributions of baryons with electric charge 2. Similarly the odd moments are modified. On the freeze-out curve this leads to a characteristic dependence of ratios of moments on the collision energy that is shown in Fig. 4. In the energy range relevant for current low-energy runs at RHIC [12] as well as at LHC one has $\kappa_Q \sigma_Q^2 \simeq 1.8$, which varies only little with $\sqrt{s_{NN}}$.

In addition one may analyze correlations between baryon number and different moments of charge fluctuations. Some results are shown in the left hand part of Fig. 4.

For completeness we show in Fig. 5 fluctuations and correlations in the strangeness sector of the HRG model. In practice it may be more difficult to compare this with experimental results as it will be crucial that the analysis allows for strangeness fluctuations in a sub-volume and will not impose the constraint of vanishing strangeness on event-by-event basis.

5 Lattice QCD and the Hadron Resonance Gas

The above considerations suggest that results on quadratic, cubic and quartic fluctuations of baryon number are in good agreement with HRG model

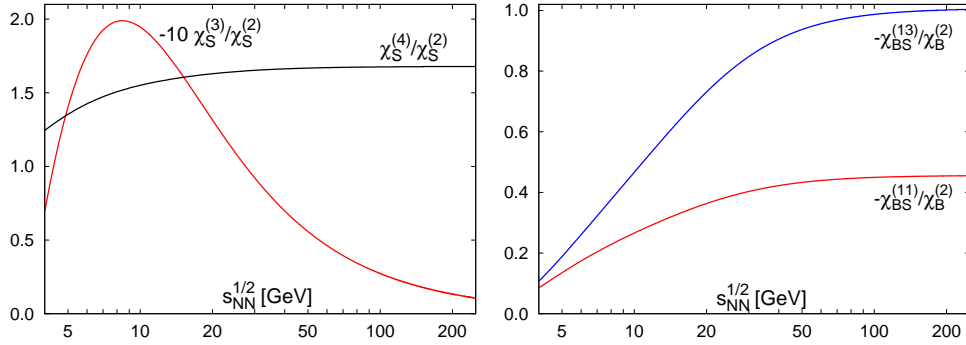


Figure 5: The ratio of moments of strangeness fluctuations on the freeze-out curve (left) and correlations between baryon number and strangeness fluctuations (right).

predictions on the freeze-out curve at least for collision energies ranging from $\sqrt{s_{NN}} = 200, 62.4$ down to 19.4 GeV. In terms of thermodynamic parameters the currently analyzed range of collision energies at RHIC covers a rather narrow temperature regime, $160 \text{ MeV} \leq T \leq 165 \text{ MeV}$. Changes in different moments of charge fluctuations with collision energy are mainly due to the variation of the baryon chemical potential along the freeze-out curve which covers the range $23 \text{ MeV} \leq \mu_B \leq 210 \text{ MeV}$.

To the extent that the freeze-out temperature for $\mu_B/T \simeq 0$ seems to be close to the transition temperature from hadronic matter to the plasma phase, the experiments cover a regime where $0.96 \lesssim T/T_c \lesssim 1.0$. In this narrow temperature range and for quark chemical potentials that are significantly smaller than T , *i.e.* $\mu_q/T = \mu_B/3T \lesssim 0.44$, there exists a lot of information on charge fluctuations from lattice QCD calculations. Due to the rather small value of μ_q/T already lowest order Taylor expansions of quark number susceptibilities provide good guidance for the behavior of susceptibilities on the freeze-out curve [16, 18, 25]. Ratios of susceptibilities have been shown to be consistent with HRG model calculations even close to the transition temperature. To this extent the results obtained at RHIC are not only consistent with HRG model calculations but are also consistent with lattice QCD results.

It may be conceivable that hadron fluctuations will only be sensitive to thermodynamics at freeze-out. One thus may ask whether fluctuations of a

completely thermalized system at chemical freeze-out in heavy ion collisions, bare any knowledge about a nearby critical point in the QCD phase diagram. In the following we will argue, that this is indeed the case.

In fact, we know for sure from lattice calculations that at vanishing baryon chemical potential and for temperatures close to the transition temperature the HRG model has to fail in describing thermal moments of sufficiently high order. This reflects the existence of a chiral phase transition in QCD at vanishing light quark mass. It is a consequence of chiral symmetry restoration that moments $\chi_B^{(n)}$ will diverge at $T_c(\mu_B = 0)$ for $n \geq 6$. At non-vanishing quark mass this leads to an oscillatory behavior of moments close to T_c ; e.g. $\chi_B^{(6)}$ will vanish at the transition temperature [18, 25]. A direct consequence of the analytic structure of the QCD partition function in the transition region is that $\chi_B^{(8)}$ will become negative. This has been observed in lattice calculations [26] as well as in chiral models [27]. In the HRG model, on the other hand, all moments of baryon number fluctuations are positive. In fact, the ratio $\chi_B^{(6)}/\chi_B^{(2)}$ will be unity in the HRG model for the same reason as $\kappa_B \sigma_B^2 = 1$. In particular at $\mu_B \simeq 0$, where the freeze-out curve is expected to be closest to the QCD transition curve or may even coincide with it, one should find strong deviations from unity. Lattice calculations suggest that $\chi_B^{(6)}/\chi_B^{(2)}$ vanishes at the pseudo-critical temperatures and rapidly rises for temperatures below, but close, to the transition temperature [25]. An analysis of these high order moments at LHC energies clearly would have the best chance to reveal differences between HRG model and lattice QCD calculations and to find evidence for critical behavior already at $\mu_B/T \simeq 0$.

Current results on $\chi_B^{(6)}/\chi_B^{(2)}$ are still noisy [25] but suggest that also this quantity may be well described by the HRG model up to temperatures close to the transition temperature. Such an agreement will stop to hold closer to the transition temperature. However, a better determination of this quantity in the transition region is necessary to quantify this and to confront it with experimental data. Eventually, it may be necessary to evaluate also $\chi_B^{(8)}/\chi_B^{(2)}$ ratio in order to observe striking differences between HRG model calculations and lattice QCD results in the energy range currently covered by RHIC and LHC experiments.

Of course, if the critical point exist in QCD and if the hadronic freeze-out occurs within the critical region, then already the second moment of baryon number and electric charge fluctuations should deviate from the HRG model result. The higher order cumulants should exhibit even stronger sensitivity

$\sqrt{s_{NN}}$	$\chi_B^{(2)}/\chi_B^{(1)}$	$\chi_B^{(3)}/\chi_B^{(2)}$	$\chi_Q^{(2)}/\chi_Q^{(1)}$	$\chi_Q^{(3)}/\chi_Q^{(2)}$	$\chi_{BQ}^{(11)}/\chi_B^{(2)}$	$\chi_{BQ}^{(31)}/\chi_B^{(2)}$
7.7	1.01	0.99	4.18	0.49	0.34	0.79
11.5	1.05	0.95	5.39	0.39	0.32	0.79
19.6	1.23	0.81	7.95	0.27	0.30	0.77
39.0	1.87	0.53	14.25	0.15	0.28	0.75
62.4	2.75	0.36	21.97	0.09	0.28	0.74
200.0	8.20	0.12	67.80	0.03	0.27	0.74
2760	111.1	0.09	922.4	0.02	0.27	0.74

Table 2: Ratios of moments of baryon number and electric charge fluctuations as well as their correlations for several values of the collision energy (in units of GeV) covering the RHIC low energy run and LHC (last row) energies. Furthermore, one has $\kappa_B \sigma_B^2 = \chi_B^{(4)}/\chi_B^{(2)} = 1$ in the entire energy range and $\kappa_Q \sigma_Q^2 = \chi_Q^{(4)}/\chi_Q^{(2)}$ varies from 1.85 at low energies to 1.75 at high energies.

to critical fluctuations showing larger deviations from the model predictions. New insight into this is to be expected to come from the first low energy run at RHIC which has been completed this year.

6 Conclusion

We have discussed properties of net charge fluctuations in nuclear matter within the hadron resonance gas (HRG) model. We have focused on the behavior of fluctuations related to baryon number, strangeness and electric charge conservation. Based on the phenomenological relation between thermal parameters and collision energy in heavy ion collisions we have calculated ratios of different moments of these fluctuations along the chemical freeze-out curve where secondary hadrons exhibit thermal and chemical equilibrium. We have also considered the energy dependence of baryon-strangeness and baryon-charge correlations in the HRG model. Our calculations covered the energy range of $\sqrt{s_{NN}} > 4$ GeV where the description of fluctuations within the HRG model formulated in the grand canonical ensemble is adequate.

Establishing generic properties of fluctuations in the HRG model we have compared its predictions with the recent data of the STAR Collaboration at RHIC on baryon number fluctuations, expressed by net proton fluctuations,

determined in Au-Au collisions at three different collision energies. We have shown, that the STAR data are consistent with the HRG model results. The change of measured fluctuations with collision energy can be accounted for by the variation of the baryon chemical potential along the freeze-out curve. A list of results from HRG model calculations at collision energies that include the published STAR results, the parameters of the 2010 low energy scan at RHIC and collision energies at the LHC are given in Table 2.

We have argued that the apparent agreement of the HRG model with the STAR data does not necessarily mean that in heavy ion collisions at chemical freeze-out the system has lost entirely its memory of the expansion period during which it may have passed through a region of the QCD phase diagram where correlation lengths are large, as expected in the vicinity of a critical point. If freeze-out occurs close to or in the transition region between hadronic matter and quark-gluon plasma this will, even in the absence of a phase transition, show up in higher order moments. The first four moments of baryon number density fluctuations agree well between HRG model and lattice QCD calculations even close to T_c . Deviations from the HRG model results, however, have been seen in higher order moments. We have argued that in order to observe critical fluctuations in heavy ion collisions one would possibly need to measure even higher order moments.

Our results on energy dependence of different moments and their specific relations in the HRG model can be used to characterize 'ordinary' thermal properties of higher order moments of charge fluctuations and to set a baseline for the observation of critical fluctuations in nuclear matter created in heavy ion collisions.

Acknowledgments

We acknowledge stimulating discussions with Tapan Nayak and Nu Xu. KR also acknowledges fruitful discussions with A. Andronic and B. Friman and the partial support of the Polish Ministry of Science. This work has been supported in part by contract DE-AC02-98CH10886 with the U.S. Department of Energy.

References

- [1] for a recent review see: V. Koch, arXiv:0810.2520 [nucl-th].
- [2] M. Asakawa, and K. Yazaki, Nucl. Phys. A **504**, 668 (1989).
- [3] J. Berges and K. Rajagopal, Nucl. Phys. B **538**, 215 (1999);
A. M. Halasz, A. D. Jackson, R. E. Shrock, M. A. Stephanov and
J. J. M. Verbaarschot, Phys. Rev. D **58**, 096007 (1998).
- [4] M. A. Stephanov, K. Rajagopal and E. V. Shuryak, Phys. Rev. Lett.
81, 4816 (1998);
M. A. Stephanov, K. Rajagopal and E. V. Shuryak, Phys. Rev. D **60**,
114028 (1999);
S. Jeon and V. Koch, Phys. Rev. Lett. **85**, 2076 (2000);
M. Asakawa, U. W. Heinz and B. Muller, Nucl. Phys. A **698**, 519 (2002);
V. Koch, M. Bleicher and S. Jeon, Nucl. Phys. A **698**, 261 (2002);
V. Koch, J. Phys. G **35**, 104030 (2008).
- [5] S. Ejiri, F. Karsch and K. Redlich, Phys. Lett. B **633**, 275 (2006).
- [6] C. Athanasiou, K. Rajagopal and M. Stephanov, arXiv:1006.4636 [hep-ph].
- [7] P. Braun-Munzinger, K. Redlich and J. Stachel, nucl-th/0304013.
- [8] A. Andronic, P. Braun-Munzinger and J. Stachel, Nucl. Phys. A **772**,
167 (2006).
- [9] P. Braun-Munzinger, D. Magestro, K. Redlich and J. Stachel, Phys.
Lett. B **518** (2001) 41;
J. Cleymans and K. Redlich, Phys. Rev. C **60** (1999) 054908;
F. Becattini, et al., Phys. Rev. C **64** (2001) 024901;
J. Cleymans, B. Kampfer, M. Kaneta, S. Wheaton and N. Xu, Phys.
Rev. C **71**, 054901 (2005);
A. Andronic, P. Braun-Munzinger and J. Stachel, Phys. Lett. B **673**,
142 (2009).
- [10] A. Andronic, P. Braun-Munzinger and J. Stachel, Acta Phys. Polon. B
40, 1005 (2009).

- [11] M. A. Stephanov, Phys. Rev. Lett. **102**, 032301 (2009).
- [12] H. Caines [STAR Collaboration], arXiv:0906.0305 [nucl-ex];
G. Odyniec [STAR Collaboration], J. Phys. G **35** (2008) 104164.
- [13] F. Karsch, K. Redlich and A. Tawfik, Phys. Lett. B **571**, 67 (2003).
- [14] A. Bazavov and P. Petreczky, arXiv:1005.1131 [hep-lat].
- [15] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti
and K. K. Szabo [Wuppertal-Budapest Collaboration], arXiv:1005.3508
[hep-lat].
- [16] R. V. Gavai and S. Gupta, arXiv:1001.3796 [hep-lat].
- [17] R. V. Gavai and S. Gupta, Phys. Rev. D **78**, 114503 (2008).
- [18] M. Cheng *et al.*, Phys. Rev. D **79**, 074505 (2009).
- [19] M. M. Aggarwal *et al.* (STAR Collaboration), Phys. Rev. Lett. **105**
(2010) 22302.
- [20] J. Cleymans, H. Oeschler, K. Redlich and S. Wheaton, Phys. Rev. C **73**, 034905 (2006).
- [21] J. Cleymans, and K. Redlich, Phys. Rev. Lett. **81**, 5284 (1998).
- [22] B. I. Abelev *et al.* [STAR Collaboration], Phys. Rev. C **79**, 034909
(2009).
- [23] A. Bazavov (hotQCD Collaboration), talk given at Lattice 2010, Vil-
lasimius, Sardinia, Italy, June 14-19, 2010;
S. Mukherjee (hotQCD Collaboration), talk given at Strong and Elec-
troweak Matter 2010, Montreal, Canada, June 29-July 2, 2010.
- [24] for recent results on the curvature in the chiral limit and a reference to
earlier results see: F. Karsch, arXiv:1007.2393v1 [hep-lat].
- [25] C. Schmidt, PoS **CPOD2009**, 024 (2009).
- [26] RBC-Bielefeld Collaboration, in preparation.

- [27] C. Sasaki, B. Friman and K. Redlich, Phys. Rev. D **75**, 074013 (2007);
B. Stokic, B. Friman and K. Redlich, Phys. Lett. B **673**, 192 (2009);
B. J. Schaefer, M. Wagner and J. Wambach, Phys. Rev. D **81**, 074013
(2010) and Acta Phys. Polon. Supp. **3**, 691 (2010);
V. Skokov, et al., arXiv:1004.2665 [hep-ph].